Sample Paper 2014: Paper 1

Question 3 (25 marks)

Question 3 (a)

> **Factor theorem**
> If k is a root of a polynomial equation $P(x) = 0$, then $(x - k)$ is a factor of $P(x)$ and vice versa.
> *or*
> For a polynomial $P(x)$, $P(k) = 0 \Rightarrow P(k) = (x - k)Q(x)$, where $Q(x)$ is a polynomial of degree one less than $P(x)$.

If $f(-k) = 0$, then $-k$ is a root of $f(x) = 0$.

$$f(x) = x^3 + (1-k^2)x + k$$

$$\begin{aligned} f(-k) &= (-k)^3 + (1-k^2)(-k) + k \\ &= -k^3 - k + k^3 + k \\ &= 0 \end{aligned}$$

Question 3 (b)

If $-k$ is a root, $(x + k)$ is a linear factor.

$x^3 + (1-k^2)x + k = (x+k)(x^2 + ax + 1)$ ← A cubic is a linear multiplied by a quadratic.

$$x^3 + 0x^2 + (1-k^2)x + k = x^3 + (a+k)x^2 + (ak+1)x + k$$

$$\therefore 0 = a + k \Rightarrow a = -k$$

$$\therefore x^3 + (1-k^2)x + k = (x+k)(x^2 - kx + 1)$$

$x^2 - kx + 1 = 0$ ← Solve the quadratic equation using the formula.

$$a = 1,\ b = -k,\ c = 1$$

$$x = \frac{-(-k) \pm \sqrt{(-k)^2 - 4(1)(1)}}{2(1)}$$

$$= \frac{k \pm \sqrt{k^2 - 4}}{2}$$

> **Formulae and Tables Book**
> **Algebra: Roots of the quadratic equation**
> $ax^2 + bx + c = 0$ [page 20]
> $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Question 3 (c)

To have exactly one real root, the quadratic equation will give complex roots. This means the expression inside the square root must be negative.

$$k^2 - 4 < 0$$

$$(k+2)(k-2) < 0$$

Solve the inequality above. First, solve the equality to locate the roots.

$$k^2 - 4 = 0$$

$$k^2 = 4$$

$$k = \pm 2$$

-3	$\boxed{-2}$	0	$\boxed{2}$	3
$(k+2)(k-2) < 0$		$(k+2)(k-2) < 0$		$(k+2)(k-2) < 0$
$(-1)(-5) < 0$		$(2)(-2) < 0$		$(5)(1) < 0$
False		True		False

Answer: $-2 < k < 2$